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The Westward Drift of the Earth's Magnetic Field

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THE WESTWARD DRIFT OF THE EARTH'S MAGNETIC FIELD

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The westward drift of the non-dipole part of the earth's magnetic field and of its secular variation is investigated for the period 1907–45 and the uncertainty of the results discussed. It is found that a real drift exists having an angular velocity which is independent of latitude. For the non-dipole field the rate of drift is $0{\cdot}18 \pm 0{\cdot}015°$/year, that for the secular variation is $0{\cdot}32 \pm 0{\cdot}067°$/year. The results are confirmed by a study of harmonic analyses made between 1829 and 1945.

The drift is explained as a consequence of the dynamo theory of the origin of the earth's field. This theory required the outer part of the core to rotate less rapidly than the inner part. As a result of electromagnetic forces the solid mantle of the earth is coupled to the core as a whole, and the outer part of the core therefore travels westward relative to the mantle, carrying the minor features of the field with it.

1. Introduction

It has long been known that many features of the earth's magnetic field show a westward drift (Halley 1692). Recently Elsasser (1949) has emphasized the great theoretical interest of this phenomenon, which deserves a more careful examination than it has yet received. Such an examination meets with two main difficulties: first, the results obtained depend on what features of the field are examined; and secondly, it is difficult to estimate how far the apparent differences between the fields at two epochs are real, and how far they are due to the incompleteness of the data and to the diverse methods employed in reducing them to maps.

The latter difficulty is greatly reduced by the work of Vestine, Laporte, Cooper, Lange & Hendrix (1947*a*). They have used virtually all the observations between 1905 and 1944 to give charts of the secular variation for 1912·5, 1922·5, 1932·5 and 1942·5. These charts have been used by them to reduce all the observations to 1945, and to construct tables and maps of all the components of the field for that year based on all the data. From this material we have computed the field for 1907·5, and our investigation is mainly based on the comparison of this and the field for 1945. In this procedure the fields compared are based on the same data and the spurious differences introduced in the reduction are as small as possible.

2. Computation of the non-dipole field

The earth's field is roughly that of a dipole with its axis not far from the axis of rotation. This predominant dipole field obscures the minor features, and it is desirable to remove its effects before looking for the westward drift. Vestine *et al.* (1947*b*) have determined the dipole for 1945 and find its moment to have components $g_1^0 a^3 = -0{\cdot}3057a^3$ parallel to the earth's axis, $g_1^1 a^3 = -0{\cdot}0211a^3$ at right angles to the axis and in the plane of the Greenwich meridian and $h_1^1 a^3 = 0{\cdot}0581a^3$ at right angles to these two directions in longitude 90° E, where a is the mean radius of the earth. The field, X, Y, Z remaining after subtracting that due to the dipole X_d, Y_d, Z_d is called the non-dipole field and is given by

$$X = X_1 - X_d, \quad Y = Y_1 - Y_d, \quad Z = Z_1 - Z_d,$$
$$X_d = -g_1^0 \sin\theta + g_1^1 \cos\theta \cos\phi + h_1^1 \cos\theta \sin\phi,$$
$$Y_d = \quad g_1^1 \sin\theta - h_1^1 \cos\theta,$$
$$Z_d = -g_1^0 \cos\theta - g_1^1 \sin\theta \cos\phi - h_1^1 \sin\theta \sin\phi.$$

Here X_1, Y_1 and Z_1 are the northerly, easterly and vertical (downward) components of the field, θ is the co-latitude and ϕ the east longitude. X_1, Y_1 and Z_1 are given by Vestine *et al.* (1947*a*, tables 49 to 51) and from them we have computed X, Y and Z. The results are given in tables 1, 3 and 5. Figure 1 shows contours of the vertical component of the non-dipole field and arrows representing its horizontal component. Figures 3 and 5 give contours of the northerly and easterly components. The contours have been drawn through points obtained by plotting the data from tables 1, 3 and 5 for meridians and parallels spaced at 10° intervals. This yields the intersections of the contours with the lines of a 10° grid and leave little arbitrariness in their form. In figures 1 and 2 the arrows representing the horizontal force always point towards areas where the vertical force is a minimum and away from those where it is a maximum. This behaviour is characteristic of a field whose origin lies within the earth.

The field for 1907·5 was obtained by subtracting the increase between 1907·5 and 1945 from the 1945 field. Vestine *et al.* (1947*a*, tables 24–35) give the rates of change at 10-yearly intervals between 1912·5 and 1942·5, call these x_1, x_2, x_3, x_4 for the X component and similarly for the Y and Z components. The field X_1', Y_1', Z_1' in 1907·5 is then given approximately by

$$X_1' = X_1 - X_v, \quad Y_1' = Y_1 - Y_v, \quad Z_1' = Z_1 - Z_v,$$

where

$$X_v = 10x_1 + 10x_2 + 10x_3 + 7{\cdot}5x_4,$$
$$Y_v = 10y_1 + 10y_2 + 10y_3 + 7{\cdot}5y_4,$$
$$Z_v = 10z_1 + 10z_2 + 10z_3 + 7{\cdot}5z_4.$$

The approximation consists in putting the change over a 10-year interval equal to 10 times the rate of change at the central year of the interval. The error depends on the third and higher differentials of the field and is of no practical importance. A similar approximation has been made by Vestine in the construction of his charts. The non-dipole field for 1907·5, X', Y', Z' is most easily obtained by computing the dipole part X_{dv}, Y_{dv}, Z_{dv} of the total secular variation and putting

$$X' = X_1' - X_d + X_{dv}, \quad Y' = Y_1' - Y_d + Y_{dv}, \quad Z' = Z_1' - Z_d + Z_{dv}.$$

The results are given in tables 2, 4 and 6 and figures 2, 4 and 6. Vestine *et al.* (1947*b*, tables 41 and 101 and p. 4) give a harmonic analysis of the secular variation from which we find the changes in the components of the dipole to be $\Delta g_1^0 = 0{\cdot}0083$, $\Delta g_1^1 = 0{\cdot}0008$, $\Delta h_1^1 = -0{\cdot}0018$. By an oversight we used in computing X_{dv}, Y_{dv} and Z_{dv}, not these values, but those obtained from an analysis of the same data using equal weights for all zones of latitude. These values were $\Delta g_1^0 = 0{\cdot}0086$, $\Delta g_1^1 = 0{\cdot}0022$, $\Delta h_1^1 = 0{\cdot}0004$. Vestine's values are to be preferred, but it was not thought worth while to repeat the work as the changes lie within the uncertainty of the data, and vary so slowly with position as to have no perceptible effect on the westward drift.

3. The westward drift of the non-dipole field

An examination of figures 1 to 6 clearly shows the westward drift. It is particularly well seen in the positions of the maxima of vertical force over the Gulf of Guinea and in southern Mongolia and in the zero line of east force off the west coast of Europe and of vertical force off the west coast of South America. On the other hand, the maximum of vertical force in the eastern United States does not appear to have moved, and there are other features that have changed so much in shape between the two epochs that it is difficult to decide whether they have moved or not.

The determination of the westward drift by picking a few conspicuous features and determining their shifts from the maps is clearly an unsatisfactory procedure. It can be slightly improved by the use of finite difference methods to find the maxima and other features. A few results obtained by this method are given in table 7. These give a rough estimate of the rate of drift (0·266°/year) and show the north-south component to be small. Since they are based on the results for selected parts of the earth, little reliance can be placed on them, and no estimate can be made of the uncertainty.

A more satisfactory method is to consider the values of the field at 10° intervals along a circle of latitude and to determine what shift in the longitude of the 1907·5 field will make it best fit the 1945 field. Let $X(\phi)$ be any component of the non-dipole field in latitude θ and longitude ϕ in 1945 and $X'(\phi)$ that in 1907·5. We now form

$$\epsilon = X(\phi) - X'(\phi + D)$$

for every 10° of longitude and for $D = 30°, \pm 20°, \pm 10°$ and $0°$. The ϵ's are the differences between the field in 1945 and that in 1907·5 shifted D degrees to the west. The most probable value of the shift is now taken as that which makes $\Sigma\epsilon^2$ a minimum, the summation being over the 36 values of ϵ^2 on a single parallel of latitude. It may be shown that this procedure is equivalent to choosing the shift that makes the correlation coefficient between the fields in 1907·5 and 1945 a maximum. Separate estimates have been made for each component of the field for every 20° of latitude, giving 27 determinations in all.

The work was entirely based on tables 1 to 6, and was done without reference to the maps. The final results therefore follow uniquely from Vestine's tables and, given those tables, are independent of any smoothing or graphical procedures. The process may be illustrated by the results for the vertical field on the equator. Figure 7 shows the fields for 1945 and 1907·5 as a function of longitude. The minimum of $\Sigma\epsilon^2$ is found by computing $(d/dD)\,\Sigma\epsilon^2$ at points half-way between the points of tabulation, and finding the value of D for which it is zero by inverse interpolation. In these and other finite difference processes fourth differences have been retained where necessary. Figure 8 shows typical curves of $\Sigma\epsilon^2$ as a function of the shift, the points at ± 5, ± 15 and 25° were obtained by interpolation.

The twenty-seven estimates of the shift are given in table 8 together with their standard errors derived in § 4 below. Since all twenty-seven are to the westward, there can be no doubt of the reality of the drift, which is established without recourse to the detailed arguments of § 4.

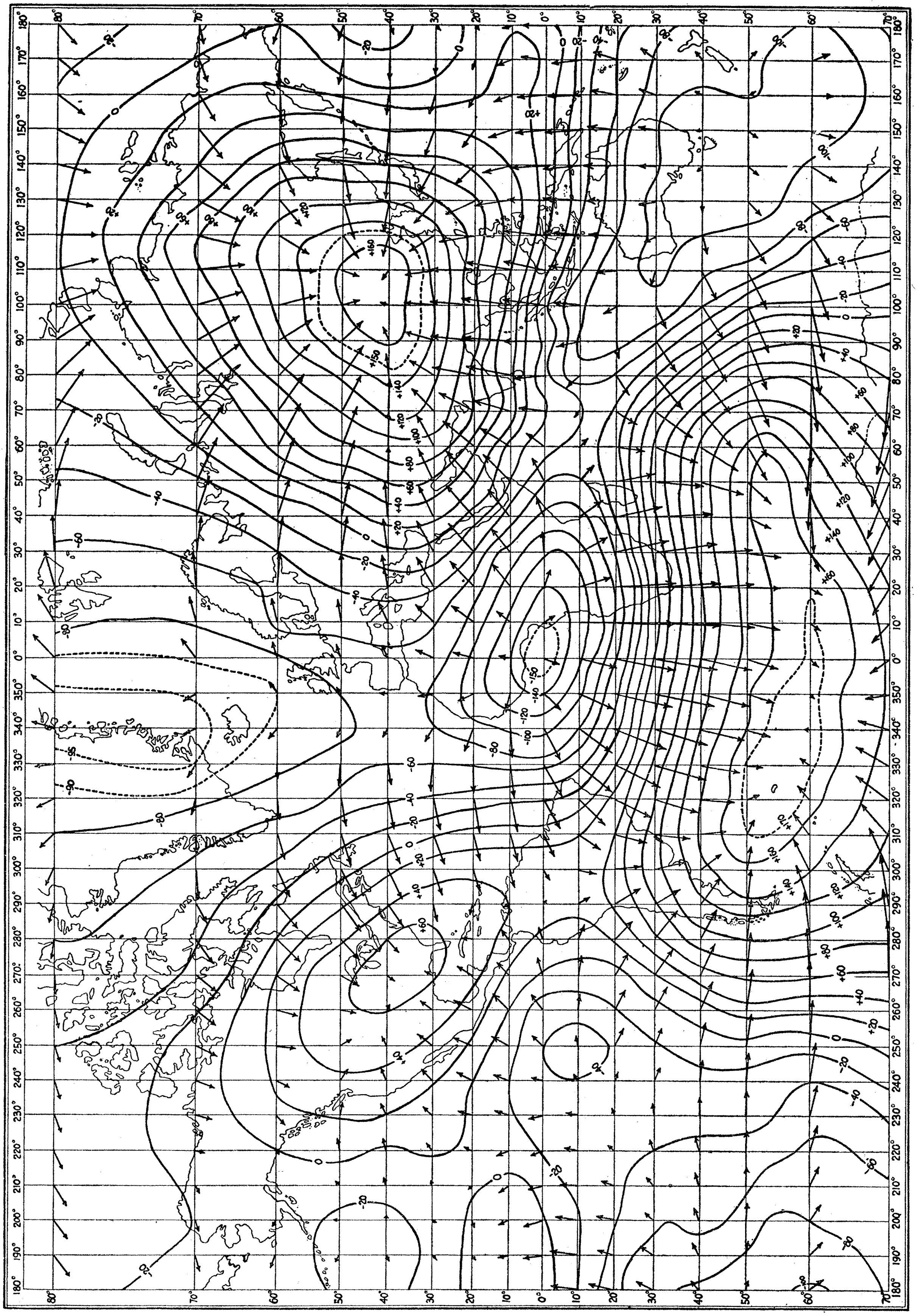

FIGURE 1. Non-dipole field for 1945. The contours give the vertical field at intervals of 0·02 gauss. The arrows give the horizontal component, an arrow 9·3 mm. long represents 0·1 gauss.

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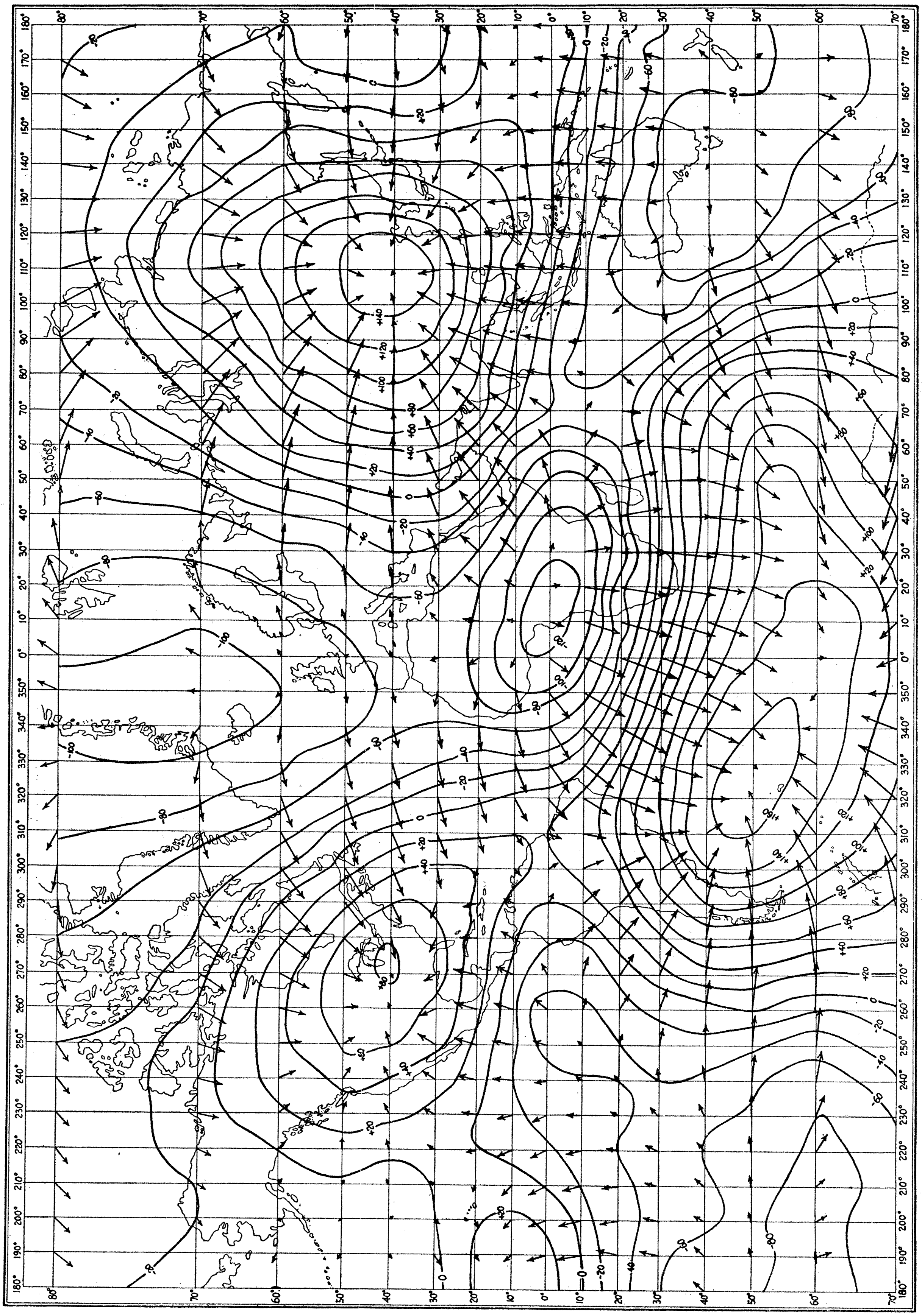

FIGURE 2. Non-dipole field for 1907·5. The contours give the vertical field at intervals of 0·02 gauss. The arrows give the horizontal component, an arrow 9·3 mm. long represents 0·1 gauss.

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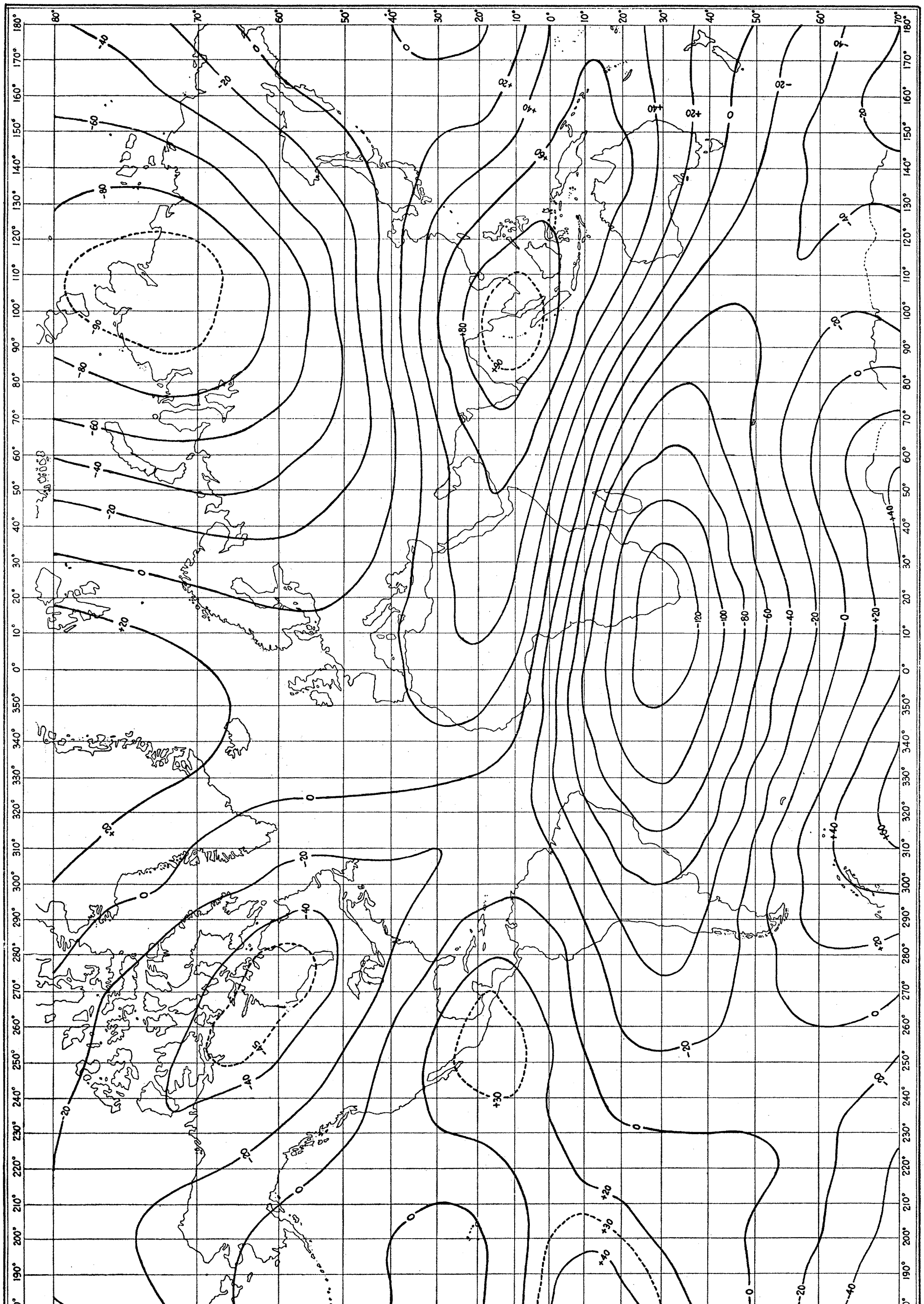

FIGURE 3. Non-dipole field for 1945, north component, contour interval 0·02 gauss.

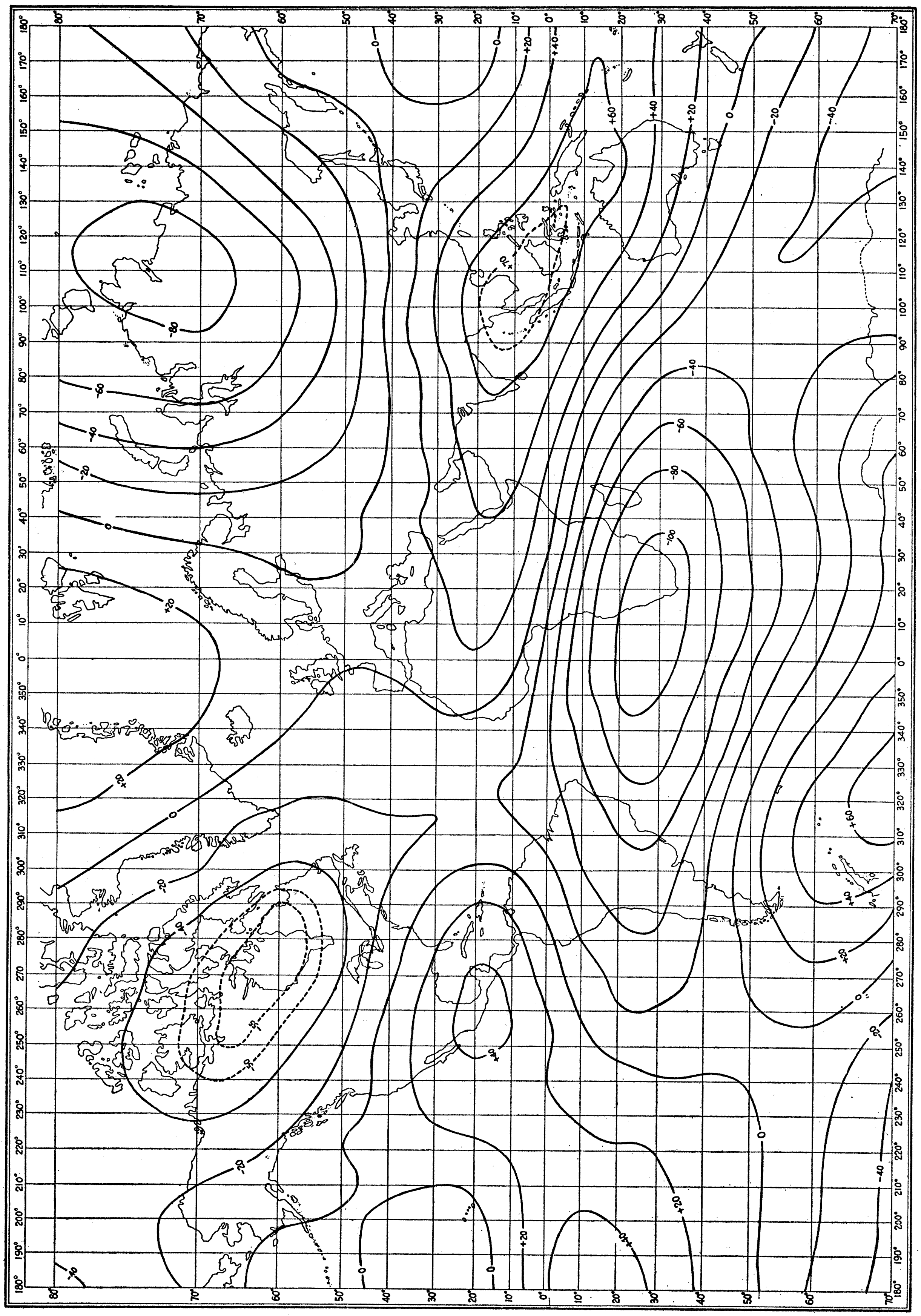

FIGURE 4. Non-dipole field for 1907·5, north component, contour interval 0·02 gauss.

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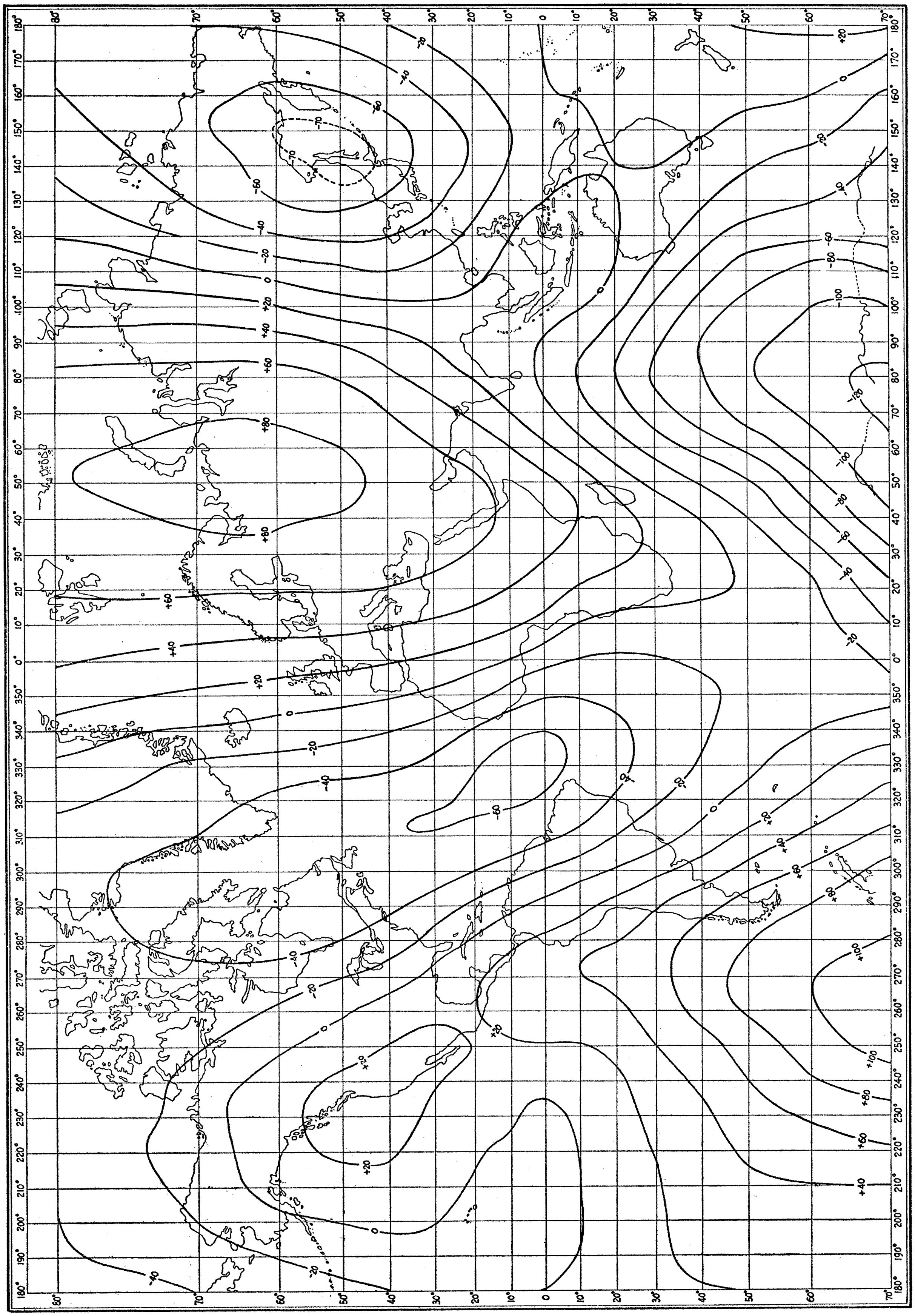

FIGURE 5. Non-dipole field for 1945, east component, contour interval 0·02 gauss.

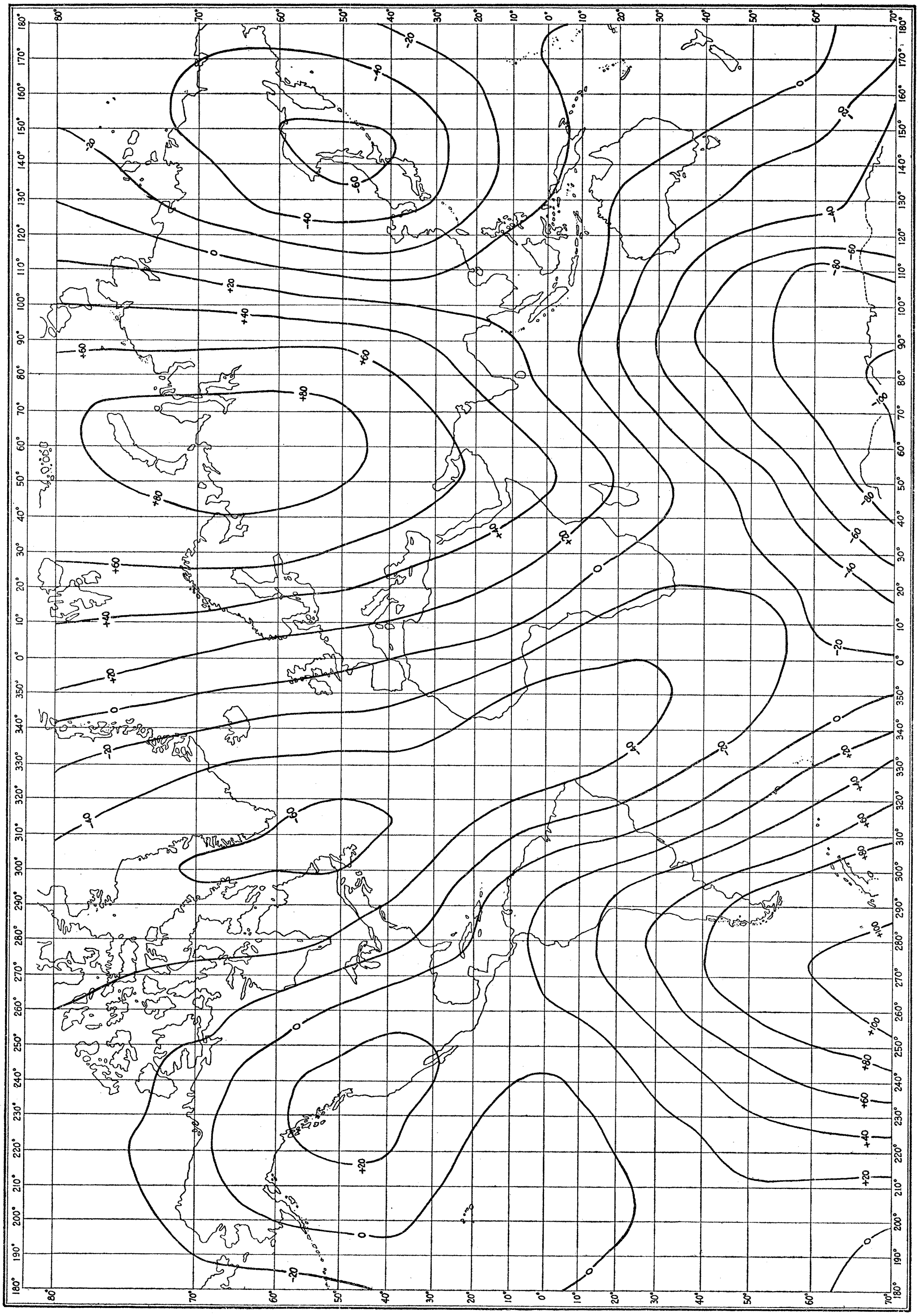

FIGURE 6. Non-dipole field for 1907·5, east component, contour interval 0·02 gauss.

TABLE 1. NON-DIPOLE FIELD, NORTH COMPONENT, 1945, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	37	22	6	10	17	34	36	15	−30	−79	−117	−127	−108	−60	−9	38	90
10°	28	15	7	7	19	36	42	25	−21	−73	−113	−130	−111	−66	−15	34	82
20°	17	5	−5	1	15	32	45	32	−12	−67	−112	−129	−110	−65	−16	35	73
30°	3	−9	−12	−8	12	34	49	40	3	−55	−105	−126	−103	−56	−12	41	61
40°	−10	−23	−25	−13	5	33	52	46	12	−42	−95	−113	−96	−49	−5	44	49
50°	−26	−40	−36	−20	1	33	56	55	21	−29	−79	−95	−82	−41	6	44	33
60°	−42	−55	−46	−31	0	34	61	64	36	−13	−55	−82	−71	−35	7	37	17
70°	−61	−69	−57	−34	0	41	69	73	50	7	−41	−67	−62	−36	2	26	−2
80°	−74	−83	−66	−37	6	46	80	85	63	22	−22	−54	−57	−40	−7	12	−27
90°	−83	−90	−73	−37	14	56	87	95	79	42	−1	−35	−49	−38	−18	−11	−44
100°	−86	−93	−76	−34	15	59	89	99	83	54	21	−15	−38	−38	−23	−30	−58
110°	−88	−93	−74	−30	14	55	82	91	85	67	38	5	−27	−38	−32	−37	−72
120°	−85	−89	−61	−27	15	47	69	81	82	68	51	21	−10	−32	−39	−38	−82
130°	−79	−79	−46	−18	13	37	55	70	75	72	57	30	−1	−25	−43	−41	−88
140°	−71	−64	−34	−9	12	26	39	55	66	71	64	34	6	−19	−45	−51	−94
150°	−64	−47	−22	0	11	19	28	40	58	69	58	35	9	−18	−45	−69	−101
160°	−55	−34	−5	5	8	8	13	31	51	62	55	34	11	−12	−40	−78	−102
170°	−48	−22	5	11	3	0	3	23	43	58	50	32	14	−11	−35	−66	−99
180°	−41	−12	13	12	−1	−6	−3	15	40	53	47	30	13	−3	−31	−60	−93
190°	−35	−10	10	9	−1	−8	−5	14	36	47	38	25	11	−2	−26	−55	−93
200°	−29	−9	4	6	0	−6	−4	13	30	38	30	18	7	−1	−20	−49	−85
210°	−26	−13	0	3	4	0	3	16	25	27	21	14	7	0	−18	−46	−78
220°	−20	−23	−10	4	7	9	13	21	24	18	13	9	5	2	−15	−43	−70
230°	−18	−34	−18	−3	10	19	22	24	22	12	2	1	0	0	−14	−38	−64
240°	−16	−40	−28	−10	11	22	28	29	18	4	−4	−10	−7	−4	−10	−31	−55
250°	−12	−43	−38	−17	6	26	35	33	22	1	−15	−16	−13	−7	−5	−23	−44
260°	−10	−41	−44	−27	−1	21	34	31	20	−3	−21	−27	−23	−10	0	−12	−27
270°	−4	−39	−48	−36	−11	10	28	26	9	−5	−29	−37	−30	−7	9	1	−6
280°	3	−33	−47	−37	−19	−1	15	17	8	−13	−34	−43	−34	−7	17	12	11
290°	8	−23	−41	−35	−24	−9	3	8	−1	−21	−42	−51	−38	−5	27	24	28
300°	18	−6	−27	−28	−24	−18	−11	−7	−11	−32	−52	−59	−44	−8	32	45	44
310°	26	−2	−14	−17	−17	−20	−15	−15	−21	−43	−64	−73	−55	−14	33	60	60
320°	33	8	−4	−6	−8	−12	−14	−16	−31	−57	−76	−86	−69	−23	26	67	73
330°	37	17	3	2	5	2	−3	−12	−36	−65	−93	−102	−80	−34	16	67	85
340°	41	24	12	6	12	14	12	−3	−36	−71	−103	−110	−92	−46	6	58	93
350°	43	24	12	8	15	27	23	5	−33	−84	−115	−119	−101	−53	−2	47	93

TABLE 2. NON-DIPOLE FIELD, NORTH COMPONENT, 1907·5, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	41	24	3	2	1	13	18	4	−34	−76	−105	−108	−82	−41	0	37	78
10°	34	20	9	3	7	17	24	13	−26	−70	−102	−110	−88	−52	−11	29	71
20°	25	13	2	2	8	16	28	20	−17	−64	−99	−107	−92	−57	−18	26	63
30°	13	3	−1	−2	9	22	34	28	−2	−51	−91	−106	−91	−54	−20	28	53
40°	1	−10	−10	−3	12	26	38	34	7	−39	−83	−94	−89	−52	−19	26	43
50°	−14	−26	−20	−8	4	27	43	41	13	−28	−70	−85	−77	−46	−11	26	30
60°	−29	−41	−31	−18	4	28	47	47	24	−17	−49	−71	−64	−38	−10	20	18
70°	−48	−56	−43	−21	3	33	52	52	34	−1	−38	−54	−51	−36	−12	12	4
80°	−62	−70	−53	−27	5	35	60	60	43	11	−21	−40	−43	−36	−17	3	−17
90°	−71	−78	−62	−31	10	42	65	71	58	29	−3	−22	−34	−33	−25	−15	−31
100°	−76	−83	−67	−32	9	44	69	78	65	41	17	−6	−25	−32	−28	−31	−42
110°	−79	−84	−68	−30	6	42	65	74	71	55	33	8	−18	−34	−36	−35	−53
120°	−78	−81	−57	−28	8	35	55	67	72	60	47	20	−6	−31	−40	−32	−60
130°	−73	−73	−43	−20	6	26	42	58	68	67	54	28	−1	−26	−41	−33	−66
140°	−67	−59	−32	−11	6	16	27	45	60	68	62	33	4	−19	−40	−41	−72
150°	−62	−44	−21	−3	5	9	16	30	52	67	57	36	9	−16	−38	−57	−82
160°	−54	−33	−5	1	1	−1	2	22	45	60	56	36	12	−8	−32	−65	−86
170°	−49	−22	3	6	−3	−9	−7	15	38	58	53	36	17	−6	−25	−53	−86
180°	−44	−14	9	7	−7	−13	−9	9	37	54	52	36	17	3	−22	−49	−84
190°	−40	−13	5	4	−6	−13	−9	11	35	50	45	33	17	4	−19	−47	−88
200°	−36	−14	−2	1	−3	−8	−6	12	31	43	38	27	15	4	−16	−44	−84
210°	−34	−20	−6	−1	2	0	3	17	28	33	31	24	15	4	−17	−44	−81
220°	−29	−30	−17	0	6	10	15	23	28	25	23	20	13	5	−16	−45	−76
230°	−28	−42	−26	−7	10	22	25	27	26	19	13	12	8	3	−16	−43	−74
240°	−27	−49	−37	−14	12	27	33	33	21	10	7	1	2	−1	−11	−39	−69
250°	−24	−53	−47	−21	9	34	43	38	24	5	−6	−5	−3	−1	−5	−32	−62
260°	−22	−52	−54	−30	4	34	45	38	21	−2	−16	−17	−11	−2	3	−21	−48
270°	−17	−51	−59	−39	−1	26	43	35	9	−8	−27	−29	−16	3	14	−6	−29
280°	−10	−46	−59	−40	−8	18	32	25	4	−20	−35	−36	−19	6	25	7	−13
290°	−4	−37	−55	−40	−15	8	21	15	−7	−30	−44	−44	−21	13	38	21	5
300°	7	−20	−42	−37	−21	−7	2	−2	−16	−40	−52	−50	−24	14	46	44	22
310°	18	−15	−30	−29	−21	−18	−10	−14	−25	−46	−60	−60	−31	14	50	62	39
320°	27	−2	−18	−20	−19	−19	−18	−21	−33	−55	−68	−70	−43	7	46	70	54
330°	33	9	−9	−13	−11	−13	−14	−19	−37	−60	−83	−84	−53	−4	36	70	68
340°	40	19	2	−8	−7	−7	−5	−12	−38	−65	−91	−91	−65	−18	23	61	77
350°	41	22	6	−3	−2	4	5	−6	−36	−79	−102	−100	−74	−29	11	49	79

TABLE 3. NON-DIPOLE FIELD, EAST COMPONENT, 1945, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	42	33	30	27	22	17	10	−1	−12	−21	−24	−21	−17	−16	−12	−24	−39
10°	52	49	45	45	42	37	31	23	13	0	−6	−7	−9	−9	−15	−39	−64
20°	62	62	62	61	58	54	48	41	30	19	10	6	4	−4	−17	−55	−86
30°	70	73	75	71	69	64	58	52	41	31	23	17	6	−7	−31	−75	−103
40°	75	82	82	78	76	69	64	56	48	39	25	13	−1	−19	−53	−96	−115
50°	78	87	89	84	76	69	62	55	47	32	11	−4	−19	−37	−72	−110	−122
60°	76	84	84	79	71	60	50	39	25	9	−10	−29	−47	−65	−91	−118	−127
70°	76	78	78	72	60	48	36	21	6	−10	−31	−50	−68	−86	−103	−122	−132
80°	64	65	65	59	45	32	19	10	−5	−19	−40	−62	−78	−96	−111	−127	−130
90°	48	48	45	37	27	17	9	5	−1	−12	−36	−56	−80	−94	−102	−116	−114
100°	31	27	20	11	5	4	4	9	11	3	−15	−41	−66	−87	−101	−104	−101
110°	13	2	−8	−16	−20	−14	−3	6	11	10	−3	−26	−52	−71	−92	−82	−96
120°	0	−23	−37	−45	−40	−29	−14	1	7	7	1	−11	−32	−51	−57	−55	−88
130°	−12	−42	−61	−63	−57	−45	−27	−10	0	3	0	−5	−16	−29	−39	−49	−73
140°	−23	−50	−66	−74	−67	−53	−35	−19	−6	−2	0	−1	−6	−19	−28	−46	−56
150°	−33	−57	−71	−70	−65	−51	−34	−19	−8	−4	2	4	1	−6	−13	−35	−37
160°	−39	−55	−64	−61	−52	−38	−25	−10	−4	1	5	12	9	5	1	−23	−19
170°	−42	−50	−51	−46	−37	−24	−14	−8	3	3	10	14	16	17	16	−2	−2
180°	−42	−40	−37	−29	−18	−12	−8	−4	0	5	11	14	19	20	20	13	15
190°	−42	−29	−18	−11	−7	−7	−5	−6	−7	1	10	17	22	28	28	19	29
200°	−41	−20	−4	0	4	−1	−8	−10	−10	−1	7	17	24	32	33	28	41
210°	−40	−13	7	8	13	5	−2	−12	−7	0	9	19	28	37	39	40	53
220°	−39	−7	11	24	22	11	3	−4	−3	1	11	22	30	43	51	56	66
230°	−39	−10	15	26	25	16	8	2	−5	4	15	26	35	52	62	72	88
240°	−37	−13	12	24	26	22	15	9	6	11	18	30	44	59	77	91	104
250°	−35	−21	−3	12	20	22	18	18	18	19	27	39	52	68	88	108	113
260°	−37	−29	−16	−3	11	17	20	25	29	30	36	49	65	82	97	119	121
270°	−37	−35	−34	−20	−8	7	17	26	31	39	44	54	70	86	98	114	117
280°	−37	−43	−47	−39	−29	−11	2	17	24	32	40	54	65	79	92	101	108
290°	−34	−46	−52	−51	−44	−31	−16	−4	6	14	28	38	53	69	81	95	98
300°	−33	−54	−54	−58	−55	−50	−41	−31	−21	−9	2	15	30	46	64	89	86
310°	−27	−40	−53	−57	−59	−58	−54	−49	−44	−35	−22	−9	8	24	43	65	73
320°	−15	−32	−43	−50	−54	−57	−62	−63	−58	−50	−39	−23	−9	7	27	47	59
330°	−3	−23	−27	−34	−36	−46	−59	−67	−66	−56	−44	−30	−18	−5	15	30	37
340°	10	−4	−11	−13	−19	−28	−41	−54	−60	−55	−44	−33	−22	−13	0	13	15
350°	31	16	11	8	3	−6	−14	−29	−39	−39	−35	−29	−24	−16	−8	−10	−12

TABLE 4. NON-DIPOLE FIELD, EAST COMPONENT, 1907·5, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	30	18	11	5	0	−4	−10	−20	−28	−36	−39	−39	−36	−28	−17	−16	−15
10°	41	35	27	22	18	13	7	1	−7	−19	−26	−30	−31	−24	−21	−30	−40
20°	53	51	47	42	34	28	23	16	7	−3	−13	−20	−19	−19	−22	−46	−61
30°	63	66	64	57	49	43	36	29	18	9	0	−7	−13	−18	−34	−64	−78
40°	71	77	76	68	64	56	50	41	32	23	11	1	−7	−23	−50	−82	−89
50°	76	85	87	81	73	65	57	50	43	33	17	4	−11	−29	−63	−94	−95
60°	77	85	87	83	75	64	55	46	37	25	11	−6	−27	−47	−75	−99	−101
70°	79	82	84	81	70	59	48	36	25	13	−3	−22	−41	−62	−81	−101	−107
80°	69	71	75	72	61	48	37	28	13	1	−18	−38	−54	−73	−88	−106	−108
90°	55	56	56	52	46	37	28	21	10	−1	−23	−42	−64	−76	−82	−98	−95
100°	40	37	32	26	22	19	18	19	16	4	−13	−35	−56	−74	−86	−89	−88
110°	23	14	5	−2	−7	−3	7	11	12	7	−8	−28	−49	−63	−80	−72	−90
120°	11	−10	−24	−32	−30	−22	−9	3	7	5	−4	−17	−33	−45	−49	−50	−88
130°	0	−29	−49	−52	−49	−39	−23	−8	1	3	−2	−8	−17	−26	−35	−49	−79
140°	−10	−37	−55	−64	−58	−45	−29	−15	−3	1	2	−1	−8	−19	−27	−50	−67
150°	−20	−45	−60	−61	−58	−45	−29	−14	−3	1	5	5	1	−7	−17	−44	−33
160°	−27	−44	−53	−53	−46	−33	−22	−8	−2	4	6	12	7	2	−7	−39	−40
170°	−31	−40	−43	−40	−33	−22	−13	−9	2	2	8	11	12	10	3	−22	−28
180°	−32	−31	−30	−25	−16	−12	−9	−7	−4	1	7	8	11	7	2	−11	−15
190°	−33	−21	−13	−8	−6	−9	−8	−11	−13	−7	2	7	10	11	7	−8	−4
200°	−34	−14	0	1	4	−3	−12	−16	−18	−10	−4	4	9	12	10	1	7
210°	−34	−8	9	8	12	2	−7	−19	−16	−10	−3	5	12	18	17	14	21
220°	−35	−4	12	24	21	7	−3	−12	−12	−9	0	9	16	26	31	31	36
230°	−36	−8	17	27	25	13	2	−6	−14	−4	5	16	24	38	44	50	61
240°	−36	−11	15	27	27	20	9	1	−3	3	10	23	37	50	63	72	80
250°	−36	−19	1	17	23	20	12	9	8	11	21	35	49	63	79	93	93
260°	−40	−28	−11	2	14	15	13	15	18	22	32	49	67	82	94	109	105
270°	−41	−36	−30	−15	−7	5	9	15	22	34	44	58	77	92	100	109	106
280°	−43	−45	−46	−36	−27	−11	0	14	23	34	46	63	77	90	98	101	102
290°	−42	−51	−54	−50	−39	−23	−4	9	22	30	45	55	70	84	91	99	97
300°	−43	−62	−60	−60	−51	−36	−17	−1	9	19	27	37	49	63	76	97	90
310°	−39	−51	−61	−63	−59	−48	−34	−21	−12	−7	0	10	25	39	55	76	82
320°	−28	−45	−56	−60	−58	−53	−49	−42	−35	−31	−25	−13	0	17	37	59	72
330°	−17	−38	−43	−47	−44	−48	−54	−56	−54	−47	−38	−28	−17	−3	20	41	53
340°	−4	−20	−29	−29	−32	−37	−46	−55	−63	−55	−46	−38	−28	−17	1	22	34
350°	18	0	−6	−11	−15	−22	−29	−41	−48	−48	−44	−41	−36	−25	−10	−1	10

TABLE 5. NON-DIPOLE FIELD, VERTICAL (DOWNWARDS) COMPONENT, 1945, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	−90	−88	−82	−69	−64	−71	−102	−140	−155	−124	−59	22	98	146	171	133	75
10°	−74	−79	−59	−56	−49	−58	−86	−129	−151	−130	−70	14	95	147	171	127	71
20°	−70	−69	−56	−37	−31	−38	−72	−109	−139	−128	−68	20	100	150	169	120	66
30°	−62	−56	−38	−18	−6	−8	−42	−87	−117	−111	−57	26	102	152	160	109	61
40°	−54	−43	−17	6	27	25	−6	−55	−89	−89	−39	45	107	157	147	96	54
50°	−44	−22	11	41	61	56	24	−23	−57	−61	−30	50	105	158	137	84	46
60°	−35	−1	50	77	93	89	48	−9	−32	−48	−21	49	101	144	121	70	36
70°	−24	15	65	102	124	112	80	21	−18	−33	−29	24	73	113	98	55	26
80°	−15	34	91	127	144	127	76	19	−34	−70	−56	−19	34	74	74	38	16
90°	−9	49	113	150	161	137	84	24	−37	−79	−86	−61	−20	28	35	16	5
100°	−6	65	125	162	169	142	88	24	−31	−75	−94	−95	−61	−22	−13	−6	−8
110°	−4	79	129	163	169	137	90	40	−16	−58	−93	−103	−86	−67	−37	−27	−19
120°	−3	78	124	152	152	128	92	48	4	−45	−82	−102	−99	−95	−62	−47	−30
130°	−3	58	109	117	118	101	79	48	11	−30	−76	−107	−109	−106	−89	−66	−37
140°	−5	38	78	84	80	70	56	40	7	−30	−75	−99	−114	−111	−96	−81	−42
150°	−8	30	51	50	37	38	34	33	8	−29	−69	−98	−110	−113	−108	−90	−44
160°	−12	15	23	16	6	10	17	24	8	−27	−65	−89	−103	−99	−109	−91	−46
170°	−17	1	6	−10	−16	−7	11	25	17	−22	−65	−86	−94	−96	−110	−86	−46
180°	−22	−9	0	−24	−25	−14	5	19	10	−20	−54	−75	−82	−87	−103	−78	−47
190°	−25	−16	−11	−29	−29	−16	4	12	9	−22	−49	−69	−72	−81	−90	−72	−44
200°	−28	−20	−12	−26	−24	−15	3	11	1	−17	−39	−57	−51	−75	−81	−67	−43
210°	−30	−18	−8	−10	−16	−6	−2	5	−8	−27	−38	−49	−51	−59	−78	−62	−39
220°	−32	−14	−2	3	−4	1	−4	−10	−20	−32	−39	−41	−47	−48	−64	−51	−26
230°	−38	−10	14	20	17	9	−4	−18	−33	−37	−35	−34	−37	−34	−53	−36	−16
240°	−37	−9	23	36	33	22	1	−22	−37	−39	−37	−30	−26	−26	−40	−16	−3
250°	−42	−15	27	51	50	34	17	−12	−39	−48	−37	−23	−9	−6	−16	9	12
260°	−48	−24	26	56	58	48	31	−3	−31	−33	−22	−7	7	30	23	33	24
270°	−53	−33	20	55	66	62	43	14	−4	−12	−7	18	46	61	59	58	35
280°	−61	−41	1	39	61	63	48	32	15	9	22	53	88	98	91	79	47
290°	−68	−48	−14	18	39	51	51	39	26	26	45	77	114	130	115	95	56
300°	−72	−57	−37	−9	16	30	31	32	27	30	52	93	134	154	138	109	63
310°	−79	−69	−55	−25	−17	−4	7	13	9	17	47	94	144	167	154	117	69
320°	−87	−83	−71	−55	−47	−36	−32	−25	−23	−11	29	85	142	171	165	123	73
330°	−93	−94	−84	−67	−59	−66	−65	−67	−65	−44	2	68	133	168	171	130	75
340°	−97	−97	−88	−82	−68	−77	−93	−104	−106	−77	−21	51	120	162	173	134	76
350°	−96	−93	−91	−79	−73	−82	−107	−134	−140	−106	−41	35	108	157	172	136	76

TABLE 6. NON-DIPOLE FIELD, VERTICAL (DOWNWARDS) COMPONENT, 1907·5, 10^{-3} GAUSS

E long. \ lat.	80° N	70°	60°	50°	40°	30°	20°	10°	0°	10°	20°	30°	40°	50°	60°	70°	80° S
0°	−98	−102	−98	−85	−72	−68	−87	−113	−121	−88	−28	42	103	137	152	112	61
10°	−83	−99	−81	−77	−62	−67	−85	−117	−129	−103	−47	23	88	129	149	108	59
20°	−79	−90	−80	−66	−56	−59	−82	−106	−127	−111	−58	12	77	122	143	102	56
30°	−72	−73	−64	−54	−40	−42	−62	−95	−115	−107	−65	3	69	117	131	92	54
40°	−63	−59	−43	−32	−15	−17	−38	−78	−104	−101	−60	12	66	117	117	81	50
50°	−54	−37	−14	3	12	8	−15	−53	−81	−81	−54	15	63	118	109	72	44
60°	−45	−17	25	39	43	40	8	−35	−54	−56	−31	24	65	109	98	62	35
70°	−34	−2	40	66	78	68	48	3	−21	−27	−24	17	53	89	83	53	30
80°	−25	17	67	95	105	91	52	11	−24	−50	−36	−11	29	62	68	40	22
90°	−19	33	92	123	130	111	69	25	−19	−48	−54	−40	−12	28	32	25	14
100°	−16	51	107	140	145	125	82	30	−12	−45	−62	−69	−46	−14	−3	8	4
110°	−13	67	114	143	152	127	88	46	0	−37	−69	−79	−69	−55	−22	−8	−5
120°	−11	66	108	136	138	119	90	53	15	−30	−64	−83	−82	−81	−43	−25	−13
130°	−11	45	90	99	104	93	76	51	20	−17	−62	−91	−94	−90	−67	−41	−22
140°	−12	27	60	67	70	65	56	45	16	−18	−61	−85	−100	−94	−72	−56	−27
150°	−14	21	38	39	32	37	38	41	21	−15	−55	−84	−96	−95	−83	−67	−30
160°	−16	8	15	11	5	13	24	36	24	−9	−47	−72	−88	−81	−87	−71	−33
170°	−20	−3	2	−11	−13	0	21	40	36	−1	−45	−68	−78	−79	−92	−69	−36
180°	−24	−10	0	−20	−18	−4	18	36	31	2	−33	−58	−69	−76	−91	−66	−39
190°	−26	−15	−7	−22	−18	−3	19	30	27	2	−31	−56	−64	−76	−86	−66	−38
200°	−29	−20	−7	−16	−10	−1	18	25	15	−4	−27	−50	−50	−77	−84	−67	−42
210°	−31	−19	−6	−4	−8	1	13	11	−1	−21	−34	−51	−58	−70	−89	−70	−41
220°	−33	−14	0	9	2	7	2	−4	−15	−29	−40	−48	−61	−67	−83	−66	−34
230°	−38	−8	17	26	23	14	1	−13	−29	−35	−38	−44	−56	−60	−80	−58	−28
240°	−36	−5	28	45	41	28	5	−20	−35	−38	−40	−40	−47	−57	−74	−44	−18
250°	−41	−10	34	64	63	42	17	−15	−44	−51	−40	−33	−28	−38	−55	−24	−6
260°	−47	−18	36	72	73	54	25	−16	−46	−45	−27	−15	−10	−2	−19	−3	4
270°	−52	−25	33	72	82	66	30	−8	−32	−32	−17	11	30	30	17	20	13
280°	−61	−33	16	59	77	66	31	2	−17	−15	8	40	66	65	50	40	24
290°	−68	−41	1	39	57	55	37	11	−3	8	36	66	98	104	78	57	33
300°	−73	−52	−25	10	36	41	27	26	14	29	63	104	137	139	106	73	40
310°	−81	−67	−47	−9	5	15	19	20	19	27	70	113	153	159	127	84	47
320°	−90	−84	−66	−43	−27	−9	−4	5	10	23	60	110	156	167	142	93	52
330°	−97	−98	−84	−61	−46	−39	−30	−27	−23	−2	42	101	151	167	151	103	55
340°	−103	−103	−96	−82	−63	−57	−59	−62	−61	−33	19	83	138	160	155	110	58
350°	−103	−103	−101	−85	−77	−69	−80	−96	−97	−64	−4	63	121	153	155	114	60

TABLE 7. WESTWARD DRIFT OF SELECTED FEATURES

place	feature	position 1945 lat.	position 1945 long.	shift 1907·5 to 1945 lat.	shift 1907·5 to 1945 long.
Gulf of Guinea	min. of vertical	0°·9 N	2°·0 E	0°·1 N	10°·6 W
Gulf of Guinea	zero of horizontal	5·6 N	2·7 E	0·5 N	8·5 W
South Mongolia	max. of vertical	43·2 N	105·8 E	0·8 N	3·8 W
England	zero of east	50 N	13·8 W	—	10·7 W
Brazil	zero of east	0	67·6 W	—	12·2 W
Turkey	zero of vertical	40 N	46·9 E	—	14·4 W
				mean	10·0 W

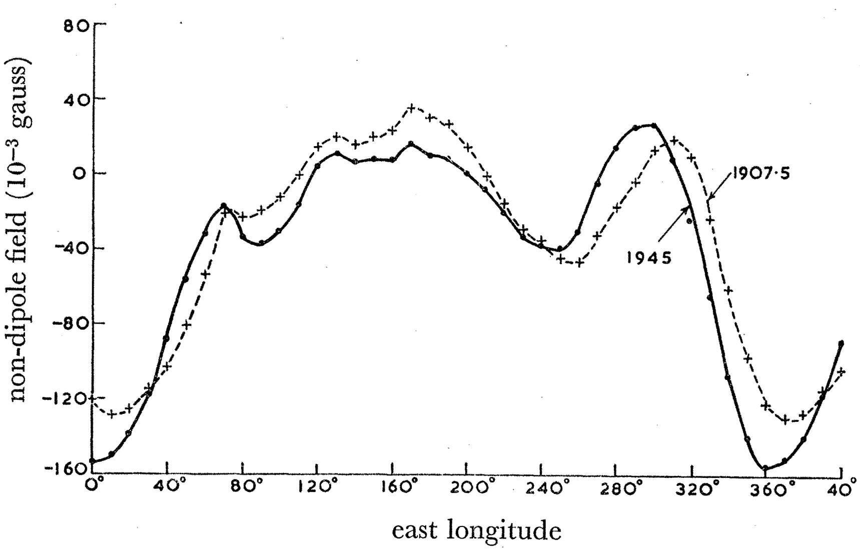

FIGURE 7. Vertical field on the equator.

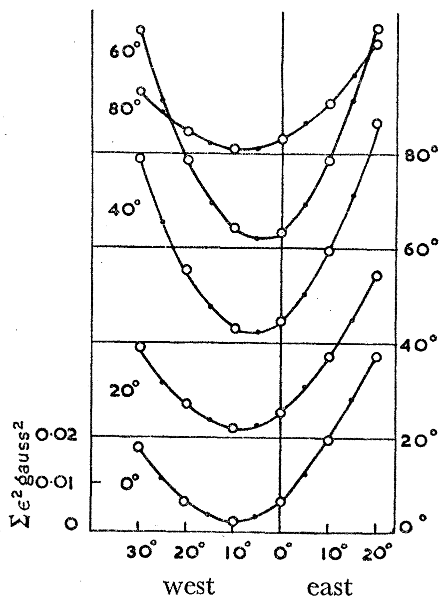

FIGURE 8. Variation of $\Sigma\epsilon^2$ with shift, east component, northern hemisphere. The figures on the right mark the zero lines for the curves referring to 0, 20, 40, 60 and 80° N latitude.

TABLE 8. WESTWARD DRIFT 1907·5 TO 1945

lat.	vertical	east	north	mean
80° N	5°·2 ± 5°·4	8°·4 ± 3°·5	8°·4 ± 4°·1	7°·8 ± 2°·4
60	4·2 ± 2·8	4·7 ± 1·6	4·2 ± 3·5	4·5 ± 1·3
40	5·6 ± 2·5	5·9 ± 1·5	11·9 ± 4·7	6·3 ± 1·2
20	9·0 ± 1·7	9·1 ± 1·7	4·0 ± 3·7	8·7 ± 1·1
0	8·9 ± 2·2	9·9 ± 1·7	4·1 ± 2·5	8·3 ± 1·2
20	10·1 ± 2·9	10·5 ± 1·9	1·6 ± 1·7	6·3 ± 1·2
40	2·8 ± 2·7	7·0 ± 2·7	6·9 ± 5·1	5·1 ± 1·8
60	6·7 ± 4·1	3·8 ± 3·4	1·5 ± 5·5	6·8 ± 2·4
80 S	13·9 ± 8·5	10·7 ± 6·0	8·9 ± 5·9	10·6 ± 3·8
mean	7·3 ± 0·9	7·3 ± 0·7	4·7 ± 1·1	6·74 ± 0·49

4. The uncertainty in the determination of the drift

The experimental errors in the determination of the magnetic field are always negligible for our purpose and the errors of reduction to the epochs 1907·5 and 1945 are usually so. The field is, however, subject to local anomalies that frequently reach 0·02 gauss. In the preparation of his charts Vestine has smoothed out all but the most extensive of these anomalies. For our purpose such a smoothing is desirable, since the local anomalies are due to the disturbance of the field by magnetic materials near the surface of the earth and have no relevance to the origin of the main field or of the secular variation. Although the smoothing is practically necessary and theoretically desirable, it does introduce an arbitrariness and uncertainty into the values at 10° intervals which have been chosen as representing the field. Thus in discussing the uncertainty of our results we have to consider not the experimental error, or the uncertainty in the reduction to epoch, but rather the uncertainty as to how far the figures in Vestine's tables represent the smoothed version of the actual field that would be obtained from a close net of stations.

If the values from tables 1 to 6 used in the calculation of the drift were all independent it would be easy to determine its standard error. However, an examination of figures 1 to 6 shows that the non-dipole field retains the same sign over distances large compared to 10°, and that neighbouring entries in the table cannot therefore be considered as independent. The irrelevance of the actual number of tabular entries that are used may be seen by considering the effect of increasing their number by interpolation. The values at 5° intervals could be interpolated between the 10° ones and the number of values used in § 3 could thus be doubled, but as no new information is introduced, it is clear that the accuracy would not be increased. There is no entirely satisfactory way of dealing with this situation. We shall assume that the 36 tabular entries used for a given latitude are equivalent for the calculation of the standard error to N_1 independent observations ($N_1 < 36$) and shall then discuss the value of N_1.

In § 3 we have used equations of condition

$$\epsilon = X(\phi) - X'(\phi + D),$$

and have chosen D to make $\Sigma\epsilon^2$ a minimum. For small shifts from the minimum we may assume at each point

$$X'(\phi + D) = X'(\phi) + D\, dX'/d\phi,$$

which gives

$$\epsilon^2 = X^2 + X'^2 + (D\, dX'/d\phi)^2 - 2XX' - 2DX\, dX'/d\phi + 2DX'\, dX'/d\phi,$$

where X has been written for $X(\phi)$. Summing and differentiating gives

$$d(\Sigma\epsilon^2)/dD = 2D\Sigma(dX'/d\phi)^2 - 2\Sigma[(X - X')\, dX'/d\phi]. \tag{1}$$

The minimum of $\Sigma\epsilon^2$ therefore occurs at

$$D = \frac{\Sigma[(X - X')\, dX'/d\phi]}{\Sigma(dX'/d\phi)^2}.$$

The change δD in D produced by a small change δX in one of the $(X - X')$ is

$$\delta D = \delta X \frac{dX'}{d\phi} \Big/ \Sigma\left(\frac{dX'}{d\phi}\right)^2.$$

If all the $(X-X')$ change independently by amounts whose root mean square value is σ, the standard deviation σ_D of D is

$$\left.\begin{aligned} \sigma_D &= \surd(\Sigma\delta D^2) = \frac{\surd\Sigma[\delta X\, dX'/d\phi]^2}{\Sigma(dX'/d\phi)^2}, \\ \sigma_D &= \frac{\sigma}{\surd\Sigma(dX'/d\phi)^2}, \end{aligned}\right\} \quad (2)$$

if δX and $dX'/d\phi$ are independent.

Since from (1) $$d^2(\Sigma\epsilon^2)/dD^2 = 2\Sigma(dX'/d\phi)^2,$$

(2) may be written $$\sigma_D = \surd 2\sigma/\surd[d^2(\Sigma\epsilon^2)/dD^2]. \quad (3)$$

Putting $\sigma^2 = (\Sigma\epsilon^2)_{\text{min.}}/N$ (3) gives

$$\sigma_D = \surd[2(\Sigma\epsilon^2)_{\text{min.}}/Nd^2(\Sigma\epsilon^2)/dD^2]$$

for the standard deviation of D assuming all the data to be independent.* If the N points used are equivalent to N_1 independent observations this expression must be multiplied by $\surd(N/N_1)$. $(\Sigma\epsilon^2)_{\text{min.}}$ and $d^2(\Sigma\epsilon^2)/dD^2$ can be estimated from the values of $\Sigma\epsilon^2$ at 10° intervals. The results are given in table 9.

TABLE 9

	minimum $\Sigma\epsilon^2$ gauss²			$d^2(\Sigma\epsilon^2)/dD^2$ gauss²		
lat.	vertical	east	north	vertical	east	north
	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-4}$	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$
80° N	11	10	12	25	54	47
60	68	20	30	198	148	57
40	131	21	16	318	135	11
20	40	20	61	170	82	54
0	84	22	21	199	88	40
20	85	22	21	122	74	85
40	121	42	69	250	89	40
60	166	52	37	235	103	28
80 S	39	76	48	36	142	92

There is no unique way of estimating the equivalent number of independent observations. In § 3 we have used 324 values of each component. The number of independent observations of a given component cannot exceed this and is probably substantially less. A good, but far from perfect, representation of the field can be obtained by analyzing it into spherical harmonics of orders up to 6. This analysis requires forty-five constants. The number of independent observations must be substantially greater than this. We arbitrarily assume 100. An uncertainty of 50 % in this only affects the standard error by about 25 %, which is perhaps as good a result as can be expected; at any rate, the value obtained should not be wildly in error and is a good deal better than no estimate. The 100 independent observations are distributed among the circles of latitude in proportion to their length. The numbers are

lat.	80°	60°	40°	20°	0°
number, N_1	3·0	8·7	13·3	16·3	17·4

The 20, 40, 60 and 80° numbers occur twice, once in the northern hemisphere and once in the southern. When account is taken of this, they add to 100. The standard errors of the twenty-seven estimates of D computed in this way are given in table 8.

* This useful expression has been used previously to estimate the uncertainty in the determination of the thickness of the earth's crust from gravity anomalies (Horsfield & Bullard 1937, p. 109).

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The above argument assumes that the ϵ's are distributed at random, apart from the correlation between neighbouring values. Their actual distribution has been investigated by finding by interpolation the ϵ's for the vertical component of the field for every 10° of latitude and longitude when D has its optimum value of 6°·74. The results agree quite well with a normal law with standard deviation 0·016 gauss, the observed and calculated numbers being:

residual (10^{-3} gauss)	−50	−40	−30	−20	−10	0	10	20	30	40	50
observed no.	0	2	46	115	109	157	96	57	23	7	
calculated no.	4	16	48	98	140	140	98	48	16	4	

The table of ϵ's is not reproduced here as it does not show much of interest. The largest residuals (up to 0·048 gauss) are due to the change in shape of the 0·160 and 0·140 gauss curves in the neighbourhood of 50° S 40° E. The next largest are due to the elongation in an east west direction of the centre in southern Mongolia (see figures 1 and 2). Such changes cannot be compensated by any general shift of the isogams.

No systematic differences are apparent between the results for the three components, and their weighted mean is therefore taken. It is not clear if the results from the three components are to be regarded as independent. They are derived from the same observations, and any one could be used to derive the other two by spherical harmonic analysis. Similarly it is uncertain how far the results from the different circles of latitude are independent. In calculating the standard errors in table 9 we have assumed that all the twenty-seven results can be regarded as independent. The weighted mean of the twenty-seven results is

$$D = 6°·74 \pm 0°·49 \text{ in } 37·5 \text{ years.}$$

The residuals from this mean are summarized in table 10. The table also gives their contributions to χ^2. None of these is remarkable except that for 20° S. As the residuals from the east and north fields in this latitude are of opposite sign this does not appear to represent a systematic difference in the angular velocity of drift. If the twenty-seven residuals in table 10 are regarded as independent and to have the relative errors obtained above, the standard error may be calculated from their mutual consistency. The result for the mean D is

$$\begin{aligned} D &= 6°·74 \pm 0°·55 \text{ in } 37·5 \text{ years} \\ &= 0°·180 \pm 0°·015 \text{ per year} \\ &= (0·99 \pm 0·081) \times 10^{-10} \text{ radians/sec.} \\ &= 20·0 \pm 1·6 \text{ km./year at the earth's surface on the equator} \\ &= 0·0344 \pm 0·0028 \text{ cm./sec. at the surface of the core on the equator.} \end{aligned}$$

TABLE 10. RESIDUALS FROM MEAN WESTWARD DRIFT 1907·5 TO 1945

	residuals			χ^2		
lat.	vertical	east	north	vertical	east	north
80° N	−1°·5	1°·7	1°·7	0·08	0·24	0·17
60	−2·5	−2·0	−2·5	0·80	1·56	0·51
40	−1·1	−0·8	5·2	0·19	0·28	1·22
20	2·3	2·4	−2·7	1·83	1·97	0·53
0	2·2	3·2	2·6	1·00	3·54	1·08
20	3·4	3·8	−5·1	1·38	4·00	9·02
40	−3·9	0·3	0·2	2·08	0·01	0·00
60	0·0	−2·9	−5·2	0·00	0·73	0·89
80 S	7·2	4·0	2·2	0·72	0·44	0·14
			sum	8·08	12·77	13·56
					$\Sigma\chi^2 = 34·41$	

The uncertainty, which is deduced from the agreement between the data for different components and latitudes, is close to that ($\pm 0^\circ\cdot 49$ in 37·5 years) found from the internal consistency of the results from the individual sets. This agreement is a valuable indication that the crude treatment of the interdependence of the data has not led to a gross error in estimating the uncertainty of the result.

The results given in the last column of table 8 and in table 10 show no change of angular velocity with latitude in excess of that to be expected from the uncertainty of the determinations. The motion appears to be a uniform westward rotation superposed on local fluctuations.

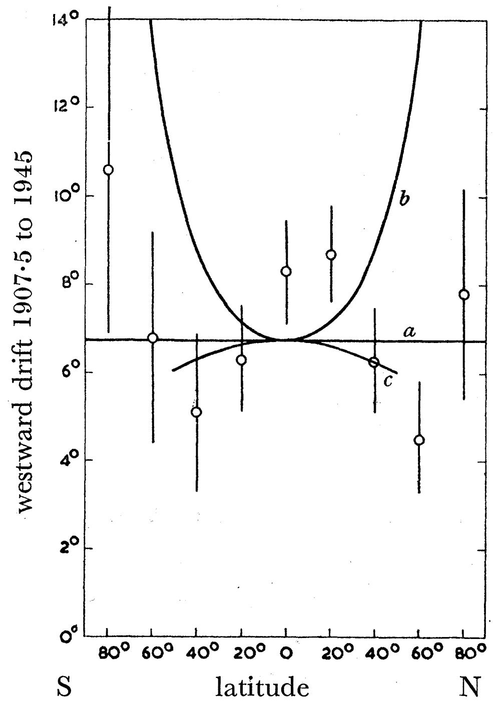

FIGURE 9. Variation with latitude of the westward drift between 1907·5 and 1945. (*a*) mean rate $6^\circ\cdot 74$ in 37·5 years, (*b*) constant linear velocity, (*c*) law of rotation of sun spots and faculae; all have been made to agree at the equator.

The constancy of the angular velocity in different latitudes is of great interest. The results are illustrated in figure 9 which shows that they are inconsistent with a constant linear velocity. The law connecting the angular velocity of sun spots and faculae with their latitude is also shown, the error in the determination of the westward drift is too great to distinguish between this law and a constant angular velocity.

It is interesting to compare our result of 0·18°/year with Halley's (1692, p. 571). He obtained 0·5°/year but remarks 'the nice Determination of this and of several other particulars in the Magnetick System is reserved for remote Posterity'.

5. Westward drift of the secular variation

Elsasser has remarked that Vestine's maps giving the secular variation for 1942·5 and 1912·5 show a westward drift. This material can also be analyzed by the methods of §§ 3 and 4. As we are now dealing not with the changes in the field, but with changes in its rate of change, the accuracy is less than that obtainable from the non-dipole field. An examination of Vestine's figures 124 to 135 (1947*a*) shows that the isopors for the north component run predominantly in an east-west direction and cannot be expected to give a good determination of the westward drift. Those for the east component are more favourably disposed. A determination could also be obtained from the vertical component but, in view of the large amount of work involved, we have confined attention to the east component. The results are given in table 11, where the standard errors are again calculated on the assumption that the data in the tables are equivalent to 100 independent observations.

Table 11. Westward drift of east component of secular variation

latitude	drift	minimum $\Sigma\epsilon^2$ $(\gamma/\text{yr.})^2$ $\times 10^2$	$\dfrac{d^2\Sigma\epsilon^2}{dD^2}$ $(\gamma/\text{yr.})^2$	χ^2
80° N	12°·5 ± 12°·2	18	8	0·06
60	0·3 ± 3·9	15	23	5·68
40	3·2 ± 3·8	51	53	2·84
20	8·3 ± 4·0	172	132	0·13
0	12·8 ± 4·6	325	180	0·48
20	15·3 ± 3·5	216	220	2·65
40	15·8 ± 4·8	231	154	1·67
60	15·3 ± 6·1	126	77	0·87
80 S	18·7 ± 10·8	123	79	0·71
mean	9·56 ± 1·6		$\Sigma\chi^2$	15·09

The results in high latitudes have, as would be expected, little weight and might have been omitted; it was, however, thought better to let them eliminate themselves from the result by their large standard errors than arbitrarily to exclude them.

All nine latitudes show a westerly drift. The values for 40° N and 60° N are low, but the errors are so large that there is no conclusive evidence for a real variation in angular velocity with latitude. The weighted mean result is

$$D_1 = 9^\circ\!\cdot\!6 \pm 1^\circ\!\cdot\!6 \text{ in 30 years.}$$

χ^2 for the nine latitudes is 15·1 which is a little high. It is possible that the secular variation data are equivalent to less independent observations than are those for the non-dipole field. If the standard error is calculated from the agreement of the results for the nine latitudes the weighted mean is

$$D_1 = 9^\circ\!\cdot\!6 \pm 2^\circ\!\cdot\!0 \text{ in 30 years.}$$

The difference between this result and that derived from the non-dipole field is

$$D_1 - D = 0^\circ\!\cdot\!14 \pm 0^\circ\!\cdot\!069 \text{ per year.}$$

It barely reaches twice its standard error. The decision whether this is to be regarded as significant rests almost entirely on the reliability of the standard error for the drift derived from the secular variation; in view of the uncertainty of this we do not consider the point as conclusively established. Six out of the nine determinations from the secular variation are greater than any of the twenty-seven determinations from the non-dipole field which suggests that there probably is a real difference. Such a difference is shown in § 7 to be possible theoretically.

6. Drift of the harmonic constituents of the field

In § 3 the westward motion of the field between 1907 and 1945 has been discussed by comparing directly the fields at these two epochs. It would be a tedious task to carry this work back to earlier periods; moreover, our knowledge of the field at earlier dates is so incomplete that it would be difficult to separate real changes from the uncertainties of interpolation.

There exists, however, another approach. Suppose the magnetic potential, Ω, to be analyzed in a series of spherical harmonics

$$\Omega = \Sigma C_{nm} P_n^m(\cos\theta)\cos m(\phi - \phi_n^m),$$

where θ and ϕ are the latitude and longitude, and C_{nm} and ϕ_n^m are constants. ϕ_n^m is related to the Gaussian constants g_n^m and h_n^m by

$$\tan m\phi_n^m = h_n^m/g_n^m.$$

A steady westward drift of the field will be shown by a linear decrease of ϕ_n^m with time. The fifteen harmonics of orders 1, 2 and 3 give six different ϕ_n^m's. These have been calculated from Vestine's data for 1907·5 and 1945 and from eight other analyses for various dates going back to 1829 (Adams 1898, pp. 133 and 135; Chapman & Bartels 1940, p. 639; and Dyson & Furner 1923). The values of the ϕ_n^m's are given in table 12 and figure 10. Where the variation with time is reasonably linear a straight line has been fitted to the 9 points. The slopes of these lines are collected in table 13, which also gives the rates of change calculated from Vestine's data only.

Table 12. Longitudes of harmonic components

date	author	dipole lat.	$180° + \phi_1^1$	ϕ_2^1	ϕ_2^2	ϕ_3^1	ϕ_3^2	ϕ_3^3
1829	Erman-Petersen	78°·3	295°·3	179°·1	137°·8	208°·7	5°·7	26°·6
1835	Gauss	77·8	296·4	182·4	135·4	201·2	8·7	31·4
1845	Adams	78·7	295·7	178·0	134·2	206·0	4·2	24·3
1880	Adams	78·4	291·9	165·8	123·8	190·1	−0·4	19·8
1885	Fritsche	78·6	292·2	165·3	122·2	194·7	0·1	18·9
1885	Schmidt	78·7	290·5	165·7	123·2	193·5	0·5	19·8
1907·5	Vestine *et al.*	78·5	290·1	159·5	112·6	193·7	1·4	13·0
1922	Dyson & Furner	78·4	290·9	157·5	105·1	195·6	3·0	5·9
1945	Vestine *et al.*	78·2	290·0	150·7	99·1	196·7	4·4	1·3

180° has been added to ϕ_1^1 so as to give the longitude of the pole in the northern hemisphere.

Table 13. Westward drift of harmonic components

n	m	westward drift °/yr. 1907·5 to 1945	westward drift °/yr. 1829 to 1945
1	1	0·003	0·062?
2	1	0·235	0·270 ± 0·016
2	2	0·363	0·341 ± 0·018
3	1	−0·080	0·113?
3	2	−0·080	0·037
3	3	0·243	0·234 ± 0·024
mean (n=2 and 3)		0·136	0·199
non-dipole §3		0·180 ± 0·015	
secular variation		0·320 ± 0·067	

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There is no evidence of any movement of the geomagnetic pole since 1880. Between 1829 and 1880 the observations suggest a westward shift of 3°·4. Such an angular shift in so high a latitude implies a linear movement of only 76 km. and, in view of the lack of movement shown by the later observations and the small amount of information available for the earlier analyses, it is considered that the observations during the last 120 years do not establish any certain shift of the geomagnetic pole. It is quite certain that its present westward angular velocity is much less than that found for the non-dipole field in § 3. The pole also shows no perceptible motion in latitude (table 12 and figure 11).

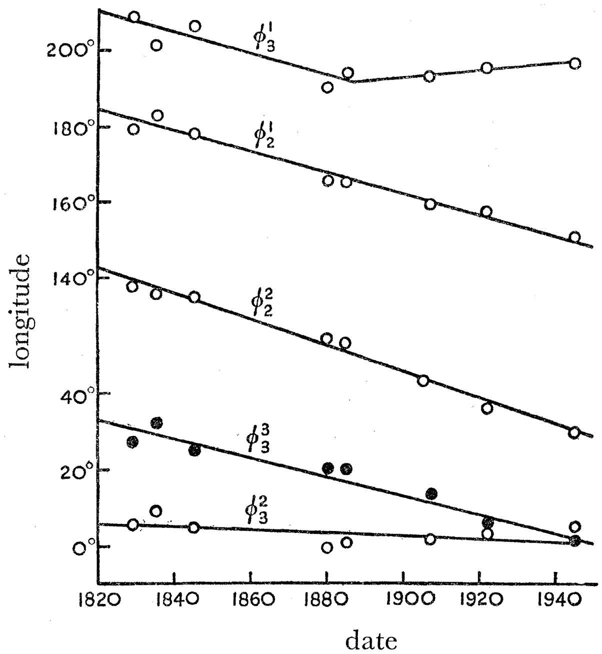

FIGURE 10. Westward drift of the spherical harmonics.

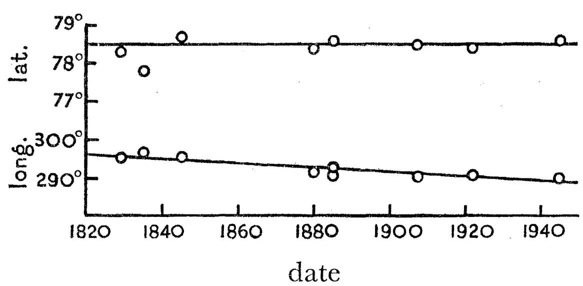

FIGURE 11. Longitude and latitude of the geomagnetic pole.

The two second-order harmonics and ϕ_3^3 show a marked westward drift of the same order as that previously found. ϕ_3^2 shows no appreciable drift and ϕ_3^1 none since 1880 (figure 10). Some irregularity is probably to be expected. The westward drift was found in § 3 to show considerable fluctuations from place to place. It is therefore to be expected that the rate of drift derived from the different harmonics will vary according to the region of the earth's surface that has most weight in their determination. These variations might be expected to exceed the uncertainties in the determination of the drift of the individual harmonics, estimated from the scatter of the points in figure 11 from straight lines. Table 12 clearly shows this, as the variation between harmonics much exceeds that implied by the standard errors of the individual entries. Actually it is likely that the latter are themselves an underestimate, as many of the measurements have been used in more than one analysis.

In spite of these difficulties and uncertainties the examination of the harmonics is of value in showing that the results derived from a comparison of the fields in 1907 and 1945 is consistent with the earlier results. The westward drift of the spherical harmonics has previously been noticed by Carlheim-Gyllenskold (1896). His perfectly genuine discovery has fallen into disrepute owing to his incorrect belief that the major part of the secular variation could be accounted for in this way, and to the absurd use of the idea by others to construct magnetic charts for remote periods. The secular change and the westward drift of the non-dipole field are related as are the changes in atmospheric pressure and the eastward motion of cyclones across the North Atlantic. The movement of the cyclones is genuine, but it does not imply the uniform motion of an unchanging system of isobars, or that the weather can be predicted far ahead. As a cyclone moves it changes in form and intensity and ultimately dies out. So it is with the centres of the non-dipole field; they move on the whole towards the west, changing in form as they go. They have not been observed over a long enough period to give a direct demonstration of their disappearance, but the continuance of the present rate of secular variation could build or destroy any of them in a hundred years.

7. The cause of the westward drift

In previous papers (Bullard 1948 and 1949 *a*, *b*), referred to as I, II and III, a theory of the origin of the earth's field has been proposed which will explain the existence of the westward drift. The treatment of it given in II is, however, incomplete, and justifiable objections have been made to it.*

It is supposed that the earth's field is produced by a self exciting dynamo. The conductor of the dynamo is the material of the liquid core of the earth, and its motion is a motion of thermal convection produced by radioactive heating. The essence of thermal convection is a radial motion, outwards in some areas and inwards in others. In a rotating sphere such a motion is inconsistent with the conservation of angular momentum, and must necessarily be combined with a radial variation of angular velocity, such that the material near the outside of the core rotates with a lesser angular velocity than that inside.

The only tenable explanation that has been offered for the secular variation ascribes it to the field produced by electromagnetic induction in material moving near the surface of the core (Elsasser 1946; Bullard 1948). The rapidity of the change in field and the restricted size of the centres of rapid change require that the motions should not be more than a few hundred kilometres below the surface of the core; if they were deeper their effects would be much reduced by screening by the overlying conducting material and would be more widespread. It is natural to suppose that the non-dipole field is largely the integrated result of the secular variation, and that its cause also lies at shallow depths in the core, though some part may be of deeper origin. These relatively shallow features will move with the outer parts of the core and will drift westward relative to the inner parts.

The above argument establishes that the minor features of the field and its secular variation may be expected to drift westward relative to the inner part of the core, but it does not establish that they will have a westward drift relative to the solid mantle. At first sight there are two possibilities, either the mantle is tightly coupled by viscous forces to the outer part of the core, or it is not. If it is there will be no observable drift, and if it is not the tidal deceleration of the mantle will cause an eastward drift. This difficulty is removed by an examination of the electromagnetic forces between the core and the mantle. Owing to the relatively low conductivity of the silicates of the mantle the forces will be much less than those between different parts of the core, but it turns out that, on any reasonable assumptions, they are far

* I am indebted to Sir Lawrence Bragg, Mr T. Gold and Dr W. Munk for their insistence that the discussion in my earlier paper was unsatisfactory. E.C.B.

larger than the viscous forces. The electromagnetic forces differ from the viscous ones in providing a coupling not merely with the outer part of the core but with the core as a whole; they therefore cause the mantle to follow not the outer part of the core, but some weighted average of the whole core. Such a coupling therefore allows the outer part of the core, and with it the non-dipole field and the secular variation, to drift westward relative to the mantle.

Calculations are in progress to find the field produced by specified motions in the core. From these it will be possible to compute the forces on a mantle of given conductivity. Until these computations are complete we must be content with cruder arguments. Let it be supposed that the core is divided into inner and outer parts of radii b and a and that each rotates like a rigid body. Let their angular velocities be ω_b and ω_a and their conductivity κ. Suppose them surrounded by a mantle of outer radius R and conductivity κ_1 rotating with angular velocity ω_1. This system departs from reality in that the continuous radial variation in angular velocity in the core is replaced by a discontinuous one. Further, no radial motion is provided, and thus the system cannot act as a self-maintaining dynamo; we therefore arbitrarily suppose a field to exist and compute the couple on the mantle. There is some latitude in the choice of field. We take a uniform field parallel to the axis and a central dipole as representing two extremes in radial variation between which the truth must lie. In fact the results do not depend critically on the radial variation of the field so long as it has its known value of about 4 gauss in the mantle near the core.

The relative rotation of the two parts of the core produces currents (S_2 currents in Elsasser's notation) flowing in meridian planes downwards near the equatorial plane thence towards the poles along paths near the axis and back towards the equator near the surface. These currents produce a toroidal field (T_2) which encircles the axis from west to east in the northern hemisphere and from east to west in the southern. All this has been thoroughly discussed in II and III. If the mantle is a conductor of electricity the S_2 currents on their way to the equator will flow partly in the mantle and their interaction with the dipole field there will produce a couple. We require to know for what angular velocity of the mantle this couple will vanish, and how long it will take it to re-establish this angular velocity if it is disturbed.

The electromagnetic problem can be solved by the methods of III; the details of the solution will not be given here as it is lengthy and uninteresting. The couple Γ is found to be given by

$$\Gamma = \frac{8}{45}\pi\kappa_1 a^5 H_0^2 \{(\omega_b-\omega_a)\, b^5/a^5-\omega_1+\omega_a\} \frac{1-a^5/\kappa^5}{\frac{2}{3}+\kappa_1/\kappa+(1-\kappa_1/\kappa)\, a^5/R^5}$$

for a constant inducing field H_0, and by

$$\Gamma = \frac{8}{45}\pi\kappa_1 a^5 H_0^2 \{(\omega_b-\omega_a)\, b^2/a^2-\omega_1+\omega_a\} \frac{1-a^3/\kappa^3}{\frac{2}{3}+\kappa_1/\kappa+(1-\kappa_1/\kappa)\, a^5/R^5}$$

for a dipole inducing field giving a field H_0 at $\theta = 0$, $r = a$.

The couple vanishes if

$$\omega_1 = (b^5/a^5)\,\omega_b + (1-b^5/a^5)\,\omega_a \quad \text{for the constant field}$$

or

$$\omega_1 = (b^2/a^2)\,\omega_b + (1-b^2/a^2)\,\omega_a \quad \text{for the dipole.}$$

If the core is divided into two parts of equal volume this gives

$$\omega_1 = 0{\cdot}32\omega_b + 0{\cdot}68\omega_a \quad \text{for the constant field}$$

or

$$\omega_1 = 0{\cdot}63\omega_b + 0{\cdot}37\omega_a \quad \text{for the dipole.}$$

The angular velocity of the westward motion of the surface of the core relative to the mantle would therefore be

$$\omega_1 - \omega_a = 0{\cdot}32(\omega_b - \omega_a) \quad \text{for the constant field}$$

or

$$\omega_1 - \omega_a = 0{\cdot}63(\omega_b - \omega_a) \quad \text{for the dipole.}$$

In practice, the constant will probably lie between these two values and for the present purpose we may take as a rough approximation

$$\omega_1 - \omega_a = 0{\cdot}5(\omega_b - \omega_a). \tag{4}$$

If the relative angular velocity of the outer part of the core and the mantle is to have the value found in § 3, $\omega_b - \omega_a$ must be about 0·36° per year. The maximum relative velocity at the boundary of the inner and outer parts of the core will then be 0·055 cm./sec. The arguments of II (p. 444) would give the corresponding radial velocity as $3{\cdot}4 \times 10^{-4}$ cm./sec. This may be somewhat underestimated as it is computed on the assumption that angular momentum must be conserved in the radial motion of every particle, in fact the electromagnetic forces tend to restore a rigid body rotation and thus increase the radial velocity necessary to produce a given differential rotation.

The maximum toroidal field is shown in III to be

$$\tfrac{2}{5}\pi\kappa(\omega_b - \omega_a)\, b^2 H_1(1 - b^5/a^5),$$

where H_1 is the field at $r = b$ (either uniform or dipole). The above value of $\omega_b - \omega_a$ gives 150 gauss for the toroidal field if $\kappa = 3 \times 10^{-6}$, $H_1 = 4$ gauss and the core is divided into two equal parts.

The above explanation of the westward drift is imperfect in that the field in the core is assumed instead of computed from the equations of magneto-hydrodynamics. Eventually it should be possible to remedy this deficiency, but it is unlikely that the order of magnitude of the results will be changed. The couple between the mantle and the core is due to the interaction of the dipole field in it with the small part of the S_2 current that leaks from the core to the mantle. This is approximately represented in the above treatment, and the main elements of the field which have been omitted (the T_2^2 fields and the S_2^2 currents of II) produce no couple.

The artificial substitution of a discontinuous for a continuous radial variation in angular velocity within the core has removed the possibility of features due to eddies at different depths moving westward with different velocities. The causes of the secular variation must lie within a few hundred kilometres of the surface of the core or their changes would be screened by the overlying conductor, but there is no reason why the non-dipole field should not possess a more permanent part whose causes lie rather deeper. In this way it seems possible to account for the westward velocity found in § 5 for the secular variation being greater than that found in § 3 for the non-dipole field.

The dipole field is believed to be due to motions extending through the major part of the core. It therefore seems reasonable that it should partake of the average motion of the whole core and not of that of the outer part only. This is in agreement with observation, but a detailed treatment would require a more thorough discussion of the motion of the terrestrial dynamo than can be given at present.

If the core does not rotate relative to the mantle with the angular velocity found above, a couple will be produced tending to restore that rate of rotation. With a uniform field the difference from the equilibrium rate will be reduced to $1/e$ in a time τ given by

$$\tau = \frac{45}{8\pi\kappa_1 a^5 H_0^2} \frac{\frac{2}{3}+\kappa_1/\kappa-(\kappa_1/\kappa-1)\,a^5/R^5}{(1-a^5/R^5)\,(1/I_m+1/I_c)},$$

where I_c and I_m are the moments of inertia of the core and the mantle. The conductivity of the mantle of hot silicate is only vaguely known. A study of induction by fields of external origin shows that at a depth of 600 km. below the surface of the earth the conductivity exceeds 10^{-11} e.m.u. If it exceeded 10^{-8} e.m.u. the secular variation would be greatly reduced by screening (I, p. 256). Extrapolation of the experimental and theoretical work of Coster (1948) suggests that the conductivity rises to about 3×10^{-9} e.m.u. at a depth of 1000 km. and then falls to about 10^{-11} e.m.u. near the core. A conductivity of 10^{-10} e.m.u. will be adopted; this corresponds to a resistivity of 10 ohm cm. Since κ is about 3×10^{-6} e.m.u. $\kappa_1/\kappa \ll 1$. The terms in a^5/R^5 and in I_c/I_m can also safely be dropped leaving a time constant of $\tau = 2D/\kappa_1 H_0^2$, where D is the density of the core. With $D = 10{\cdot}7$ g./cm.^{3}, $H_0 = 4$ gauss, this gives 10^{10} sec. or 300 years. A similar result is obtained if the couple appropriate to a dipole inducing field is used.

If the tidal deceleration of the mantle is $\dot{\omega}_1$ radian/sec.^{2} it may be shown that the westward drift will be reduced by $\dot{\omega}_1\tau$. With $\tau = 10^{10}$ sec. and $\dot{\omega}_1 = 2\times10^{-22}$ sec.$^{-2}$ this gives 2×10^{-12} sec.$^{-1}$ (0·004°/year) which is negligible. The coupling between the mantle and the core is therefore sufficiently tight to prevent the tidal deceleration of the earth from appreciably reducing the westward drift. If the conductivity of the mantle were a hundred times less than we have assumed, this would no longer be true; the specific resistance of the mantle would then be over 1000 ohm cm., which is certainly too high.

If the mantle rotates with the angular velocity given by (4) no current flows in it. If its angular velocity departs from this by an amount $\delta\omega$ the maximum value of the current density is approximately $\frac{1}{2}\kappa_1 a H_0 \delta\omega$. If $\delta\omega$ were such as completely to remove the westward drift, that is if the mantle moved with the outer part of the core, this would give $1{\cdot}4\times10^{-12}$ amp./cm.^{2} in the mantle. This is small compared to the currents in the core which are about $\frac{2}{5}\kappa b H_0(\omega_a-\omega_b)$ or approximately κ/κ_1 times as great.

It might be suggested that the observed fluctuations in the rate of rotation of the earth are due to electromagnetic coupling of the mantle to turbulent motions in the core. The above results show that this cannot be so, since the time constant of such processes will be at least as great as the 300 years calculated above, whilst the observed changes sometimes occur in a few years.

It remains to show that the neglect of viscous forces is justified. If the mantle starts to rotate relative to the outside of the core with angular velocity ω, a boundary layer will be formed in which the velocity changes from that of the mantle to that of the core. This boundary layer will at first be thin compared to the radius of the core. Let its thickness be s. The viscous couple between the mantle and the core will be $\pi^2\omega\eta a^3/s$, where η is the viscosity. In order to supply this couple from electromagnetic forces the relative angular velocity of mantle and core must depart from the value given by (4) by an amount $\delta\omega$ given by

$$\frac{\delta\omega}{\omega_1-\omega_a} = \frac{10\pi\eta}{s\kappa_1 a H_0^2}.$$

Putting $\eta = 10^{-2}$ g./cm.sec., $\kappa_1 = 10^{-10}$ e.m.u., $a = 3{\cdot}5 \times 10^8$ cm. and $H_0 = 4$ gauss this gives

$$\frac{\delta\omega}{\omega_1 - \omega_a} = \frac{0{\cdot}06}{s}.$$

The viscous forces are therefore powerless to alter the westward drift appreciably even if the boundary layer is as thin as one centimetre. The same conclusion may be reached by considering the time required for viscous forces alone to reduce the relative angular velocity of core and mantle to $1/e$ of its initial value. This time is approximately $sDa/5\eta$ or $2 \times 10^3 s$ years (s in cm.). For any reasonable value of s this is much greater than the corresponding time for the electromagnetic forces, and for very moderate values of s may exceed the age of the earth.

From the above discussion it appears that the dynamo theory requires a westward drift of the non-dipole part of the earth's magnetic field and of the centres of secular variation, and that the observed rate of drift can be accounted for by reasonable values of the quantities concerned. The westward drift gives the most direct estimate of the velocities in the core, and it is encouraging that the result obtained is of the same order of magnitude as that deduced from the secular variation (II, figure 2). An independent estimate can be obtained by computing the critical velocity necessary for self excitation of the dynamo. An investigation of this is in progress.

Whilst the above explanation of the westward drift grew out of the dynamo theory, it would also be consistent with other theories of the origin of the main field, provided the differential rotation of the core occurs. The explanation uses most of the mechanism of the dynamo, but does not exclude the possibility that, in spite of the existence of this mechanism, the field might be produced by some other means.

Elsasser (1947, 1949) has suggested that the westward drift is to be explained by a slowing down of the rotation of the core by tidal forces. Reasons for supposing this effect to be negligible have been given in II, p. 438. K. Runcorn and W. Munk have made unpublished suggestions that the westward drift is due to a westward motion of eddies in the core through the general body of fluid. This idea is worthy of detailed investigation; in particular it would be interesting to know what variation of drift with latitude would be expected.

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